Metrology of X-ray Optics for Astrophysical Applications

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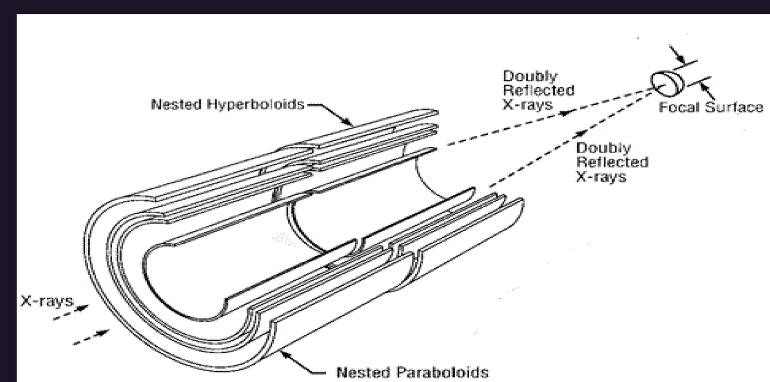
Introduction

The study of X-rays has proved to be a valuable asset to the discovery and understanding of the origins of the universe. In order to view X-rays that are scattered throughout space, specialized optics must be used that reflect the X-rays at grazing angles, such as the optics on the Chandra telescope. These optics are a combination of parabolic and hyperbolic reflectors which reflect incident X-rays to a common focus. Because the X-rays must be reflected at small grazing angles, the surface of the optics must be incredibly precise. In order to obtain such a high precision, the optics must be deposited with another material such as nickel to flatten out the surface, and improve resolution, in this case, nickel was the main material used to coat the optics. The problem that arises with the coating process is that the added thickness of the coating will cause stress on the optics which then bend and change from an ideal geometry. To predict what kind of change the optics will undergo, mathematical models using Stoney's equations and based on the equations defining Wolter geometries are utilized to obtain the ideal parabolic and hyperbolic optics. As the models replicate ideal optical performance, stress values were added to the Wolter prescription to examine the deformations occurring on the optics. Different coatings and stress values obtained from experimental results were tested in the models to determine the effects of each on the radius of curvature of the ideal optics. After the predicted displacement was determined, we used the Finite Element Analysis (FEA) software COMSOL Multiphysics to obtain visual models that would replicate the displacement as described by Stoney's equations.



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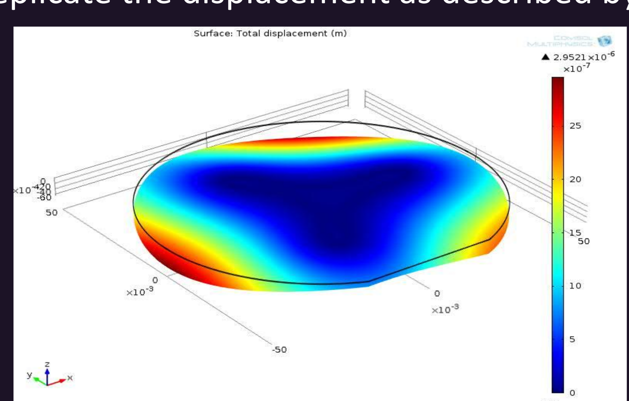


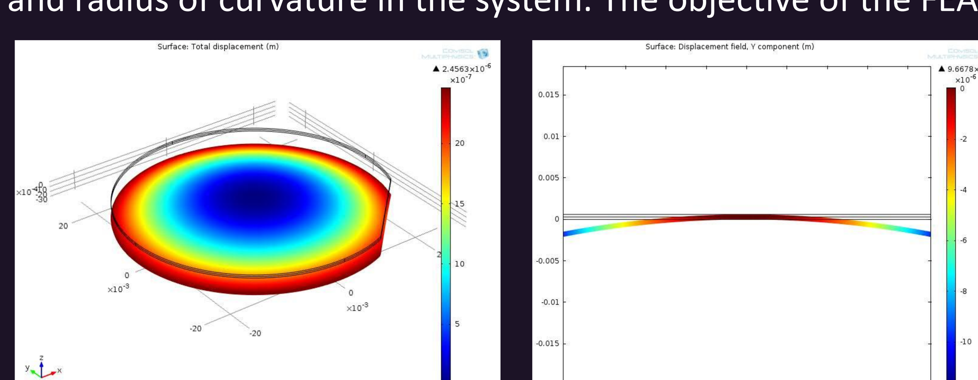
COMSOL Multiphysics is a FEA software package that is used to build 2D and 3D models and then apply physics to the model and observe the results. The picture to the left shows a silicon wafer with the effects of gravity modeled on it.

Methodology

Multiple experiments were run to obtain a range of stress values that the nickel coating caused on the optics. The experiments involved setting up a vacuum chamber with a nickel plate and then using RF (radio frequency) sputtering with ionized argon atoms which are attracted to the nickel target, and knock off some nickel atoms, which are then sputtered onto the desired surface.

Once the displacement was measured, it was entered into a spreadsheet that used Stoney's equations to obtain the stress values and calculate the radius of curvature caused by the coating. Using Stoney's equation a ROC was derived from the experimental stress values and a given uniform film thickness. This ROC was then added to the parabolic and hyperbolic form to observe how the stress would affect the resolving capability of the optic. Finite element analysis (FEA) was performed to assess the applicability of Stoney's equation for stress evaluation in the coating system. The analysis used thermal interactions between the coating and substrate to directly emulate the displacement observed from the Stoney's equation. The residual displacements thus estimated by FEA, equation were observed to be in good agreement with those computed by FEA, suggesting that the FEA thermal approximation can be used to estimate displacement and radius of curvature in the system. The objective of the FEA research was to obtain a measure of the errors induced in the optical form by stress in the coating layer - half power diameter (HPD) was used to quantify the errors.

Inside of vacuum chamber showing nickel plate and plasma deposition on a brass shim.



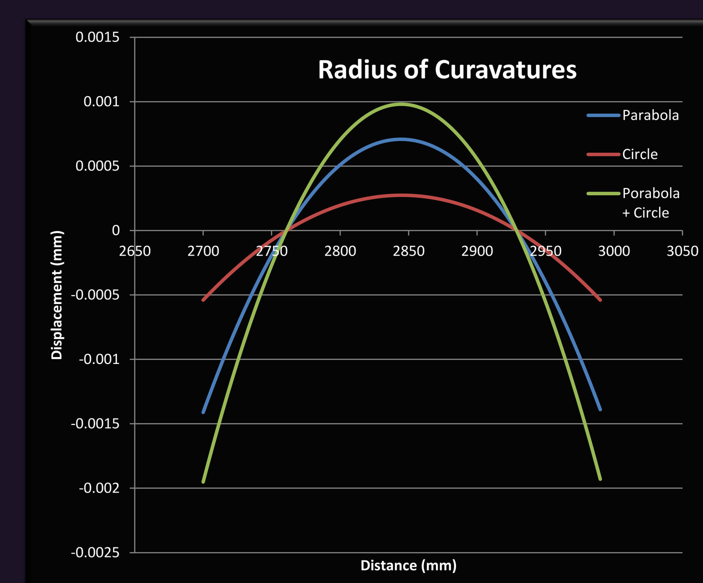
Examples of 3D and 2D silicon wafer FEA modeling using thermal interactions showing the displacement caused by a coating. Both were coated with nickel.

Modeling of Radius of Curvature

The radius of curvatures of the paraboloid (rp) and hyperboloid (rh) were modeled using the Wolter equations presented in VanSpeybroeck and Chase's paper, they are as follows respectively.

$$r_p^2 = P^2 + 2PZ + \left[\frac{4e^2Pd}{e^2-1}\right] \qquad r_h^2 = e^2(d+Z)^2 - Z^2$$

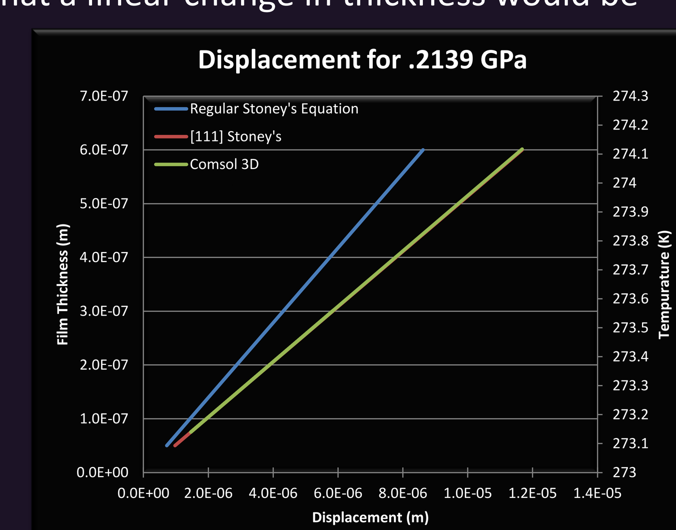
Once the parabola and hyperbola were modeled on a graph, the equation of a circle including the achieved stress values from experimentation was added to the same graph. The equation of a circle represents what the theoretical stress value would do. After the stress values were graphed onto a circle properly, the paraboloid and hyperboloid functions were added to the circle equations to get a final prediction as to how the stress of the coating would effect each one.



In the graph you can see the ideal curve in blue, this shows what the ideal parabola should look like to obtain optimal focus. The red line shows how the stress will displace at a film thickness of 2.0X10^{-5}mm, a substrate thickness of .5 mm and a stress value of .05 GPa. The two curves were then added together to predict what the radius of curvature of a coated parabolic optic would look like. These initial results had displacement on the order of .0001 mm.

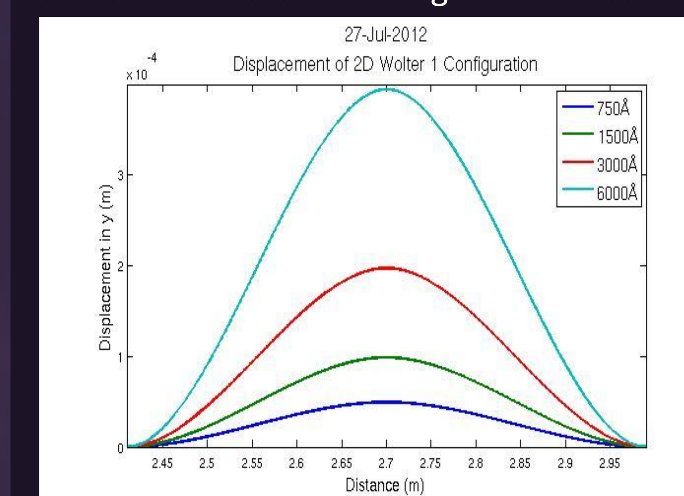
Validation of FEA modeling.

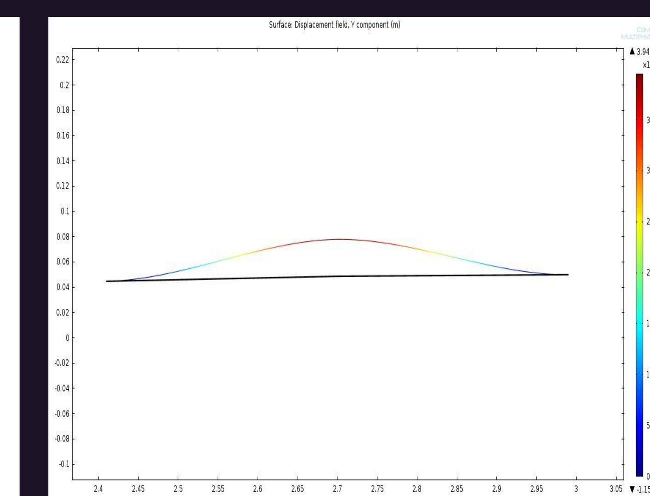
Once the models of the 2D and 3D silicon wafers with a coating were obtained, the thermal expansion was simulated to predict displacement from coating. The displacement predicted by the FEA was then checked against the displacement predicted by Stoney's equations to check that a linear change in thickness would be equivalent to a linear change in temperature. The graph shows predicted displacement for regular Stoney's equations (blue) and Stoney's equation for silicon [111] (red), A variant of the Stoney's equation which takes into account the anisotropic nature of the silicon wafers. As shown, the COMSOL FEA displacement for silicon [111] (green), which is over the red line, Linearly deforms in accordance with the Stoney's equation



2D Wolter FEA Model

Using COMSOL Multiphysics, a conical Wolter geometry approximation was modeled in a 2D setting and a coating was placed on the inside edge to run the simulation. Using the data obtained from the 2D silicon wafer FEA model, the same temperature difference was applied to the 2D Wolter case in order to obtain a predicted displacement. Four different temperature differences were applied to the FEA model to simulate 4 different coating thicknesses.

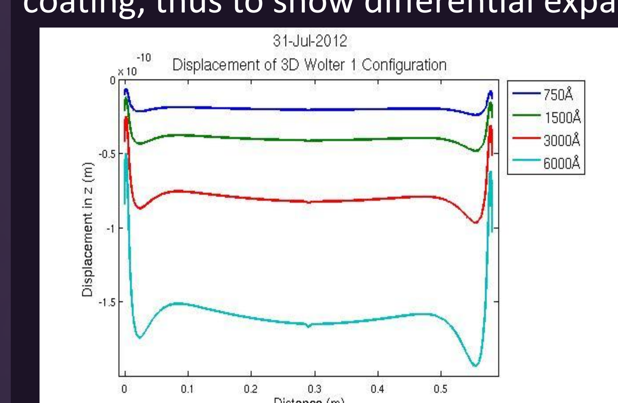


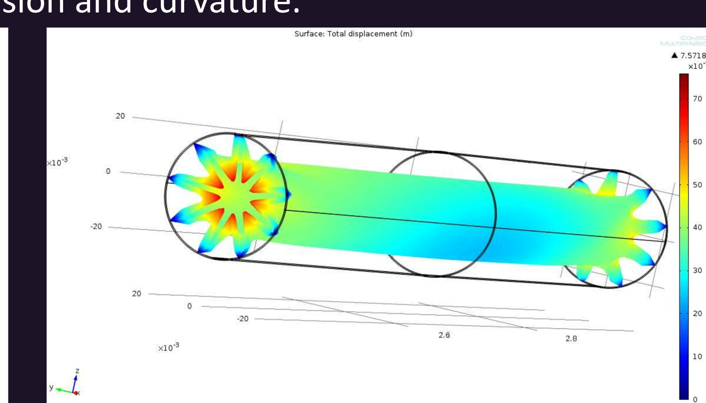


The graph on the left shows the displacement for the 4 different thickness simulated, while the picture on the right shows what the FEA model and analysis looked like. It is important to note that the 2D model only take a small section into account and does not represent a 3D structure and therefore lacks in rigidity, thus the larger displacement then the 3D model.

3D Wolter FEA Model

The Wolter geometry was modeled in a 3D cylindrical setting and a coating was placed on the inside of the model. Using the data obtained from the 3D silicon wafer FEA model, the same temperature difference was applied to the 3D Wolter case in order to obtain a predicted displacement. Four different temperature differences were applied to the FEA model to simulate 4 different coating thicknesses. Both the model and the coating were nickel and therefore the temperature difference was only applied to the coating, thus to show differential expansion and curvature.





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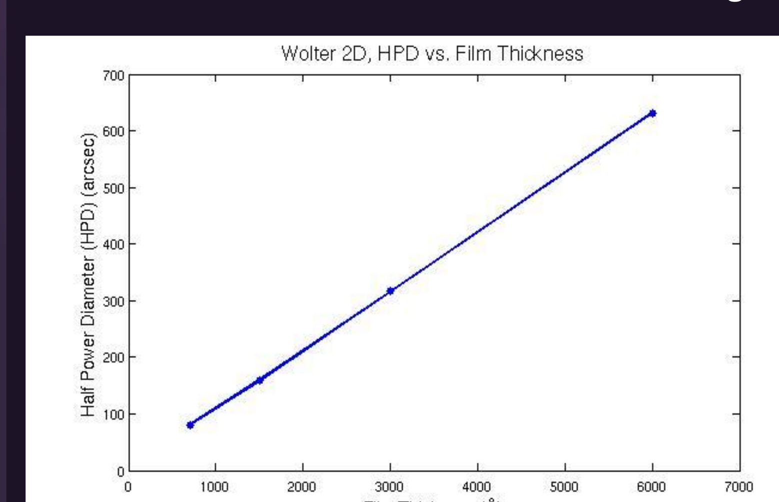
Half Power Diameter (HPD)

After the displacements have been calculated, it is important to determine how the deformations will affect the resolution of the optics. Therefore the HPD was calculated for the 2D COMSOL model deformations using a slope sort method.



The starting HPD of the Wolter configuration was approximately 12 arc seconds. By adding even a thin layer of nickel, 750A, the HPD was significantly degraded. However, it is important to note, that while this is well out of the range of what is needed to observe X-rays, this is just for a 2D model. A 3D model would have far less displacement because of the solid structure of the model.

Future Work

- Use the displacement calculated from the 3D Wolter FEA model to obtain an half power diameter of the 3D system.
- Simulate different coating properties and thicknesses on the FEA models.
- Use the theoretical results in a finite element analysis (FEA) to more accurately predict what the stress will do to the optics and allow other forces to be implemented on the parabolic and hyperbolic optics.

Conclusion

The radius of curvature graph in this study showed the theoretical differences between the ideal parabolic curve and the displacement cause by the added stress of the coating. It is important to quantify the deformations and the effect on the HPD to determine the success of the differential deposition process when correcting for surface errors. In addition, COMSOL was shown to be able to accurately represent the curvature of Stoney's equation as a thermal representation. By using the FEA to further characterize the problem, we can then in turn can use it to estimate what the half power diameter will be. The results from the half power diameter show how the optics are able to perform and see what type of resolution error will be received in the focus of the optics. Knowing this will allow future designs to adjust accordingly to obtain a desired focus for a range of different values.

References

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